ACTUARIAL STATISTICS

By

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The subject of actuarial statistics can be more readily understood if we first define what an actuary is, and describe the nature of his work.

Contrary to popular impression, the actuary did not originate as a mathematician. From the beginning of modern life insurance, which commenced in England some 200 years ago, the actuary has always been identified with the management of insurance funds, and his qualification as a mathematician came later as actuarial science developed.

Definition of Actuary

The term "actuary" has its origin in the Latin word actuarius, meaning a registrar or clerk. This title appeared in history as an officer attached to ecclesiastical courts with the duties of registrar. In Roman times, the actuarius recorded the acta of Senate.

The title of actuary, including duties as a registrar, first appeared as a life insurance officer in the deed establishing the Equitable Society in England in 1762, the first modern life insurance company. Since then, there have been many definitions of actuary, mainly because of the changing, or continuously enlarging, scope of his activities and duties. Therefore, a definition of actuary has to be flexible, all-embracing, and in a sense, temporary.

For our purpose today, for the definition of an actuary, we quote from, and take the liberty to paraphrase, the address of the President of the Society of Actuaries in the United

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States, Mr. Edmund McConny. Mr. McConny stated, in November of 1949, at the formation of the present Society of Actuaries in the United States, that "an actuary in reality is a sound, practical — rather than too theoretical — mathematician applying simple principles of probabilities to human affairs in the unknown future." He applies the facts he gathered to such unknowns as future mortality and sickness rates; future interest earnings under governmental controls and free enterprise; future expenses under continuous inflationary tendencies; and future human characteristics in regard to security, savings and family needs.

Although in practice actuaries deal with both life and non-life insurance, it was primarily the problem of life insurance that brought about the actuary. Thus, in common usage, the term actuary refers exclusively to that of a life insurance actuary.

Nature of Actuarial Work

Although the present day actuary is concerned with every phase of life insurance operations, his main responsibilities comprise the following areas:

(1) Calculation of premium rates, reserves, non-forfeiture and other related policy values;

(2) Underwriting or selection of risks;

(3) Investment and asset valuations;

(4) Studies of company costs and other expense factors;

(5) Preparation of long-term projections to guide overall company operations; and

(6) Provide adequate safeguard against catastrophic losses by means of reinsurance facilities.

The basic tools of an actuary are the mortality table and compound interest functions. The mortality table is the scientific basis of all life insurance contracts. By means of it, the probabilities of living and dying can be ascertained, on the assumption that what has taken place in the past will be approximately reproduced in the future.

A mortality table consists of a schedule showing for each age the number of persons who die and the number who survive out of a known number under observation. The construction of a mortality table, or other similar table of sickness, marriage, withdrawal, etc., was the original, and still is one of the most important, tasks of the actuary. It is essentially a work involving the choice, collection, analysis, and presentation of statistical facts.

The application of compound interest involves more than simply the utilization of established formulas. It requires an intimate knowledge of the securities market, investment returns, and valuation of assets. The actuary is called upon to estimate average investment yields over a long term period, usually 50 or 60 years. For this work, he needs to be familiar with the economic conditions of the country, as well as the monetary and fiscal policies pursued by the government.

The other works of the actuary demand that he be possessed of a variety of knowledge. In underwriting or selection of risks, he has to be a good judge of human behavior and character. He has to appraise the desirability of the applicant for insurance purposes from the standpoint of morals, physical conditions, and financial stability. In studies of company costs and other expense factors, the actuary does the work of a cost accountant and comptroller. In preparing long-term projections, the actuary helps plan out the long-term sales objectives of the company. In providing adequate safeguards against catastrophic losses, he has to negotiate for suitable and reasonable reinsurance terms with other insurance companies.

Actuarial Statistics

In his work, the actuary has need of a great variety of statistics as well as statistical techniques. He makes use of standard methods of sampling, measures of central tendencies, frequency distributions, correlation, graduation, interpolation, extrapolation, and testing of hypothesis.

Most of the statistical work of an actuary are related to mortality; thus mortality statistics is the principal preocupation of an actuary.. The mortality table is first introduced to the student of actuarial science as a finished product by which he can calculate probabilities of death and survival, and therefore derive the necessary monetary values. The process of constructing mortality tables, however, presents a practical problem in statistics.

The earliest mortality tables were those used by the Romans for determining the values of life estates. Under the Falcidian law, a man could not bequeath more than three-fourths of his property away from the real heir; and it was a common practice to bequeath annuities or life interests, the values of which had to be determined by computation. It is interesting to note that, one of these tables, giving values of life expectations, and the present value of life annuities, although prepared in the third century, continued to be used officially in Northern Italy until the end of the 18th century.

The first scientific mortality table was developed in 1693, by the famous astronomer Halley, of comet fame. This table was derived from a study of births and deaths that have been recorded for a period of five years in the parish registers in a town in Silecia. Since then, many mortality table have been developed and used for various purposes.

For the past hundred years or so, the tables which have been most valuable for insurance purposes have been derived from the records of life insurance companies. These tables are referred to as "experience tables" or "insured tables," in contrast to "population tables," the sources of which were population statistics. It is in the area of "experience tables" that the actuary has shown greater interest, and, of course, done more work.

In the construction of the mortality table, we have one of the best examples of "actuarial statistics" at work. We will, therefore, describe briefly the basic steps involved.

The first step is to have an objective. Why do we want to construct another mortality table?

The answer to this question is the ever changing pattern of mortality. Although in some investigations, the progression of the rates of mortality from age to age has been found to conform closely to one mathematical fomula or another, such as those of Gompertz and Makeham, no law has yet been discovered which is universally applicable. The many underlying factors affecting mortality — age, sex, occupation, heredity, year of birth, residence, and environment — are so complex that their influence on mortality is constantly changing the character of the mortality rates.

The frequency of constructing mortality tables depends not only on the availability of reliable statistics but also on the time lapse — the length of time that has passed since the last useful table was constructed. Obviously, it is not wise to attempt work on a new table without sufficient data. Likewise, it will not be necessary to have a new table if the current table was constructed only "recently" — "recently" taken to mean that during this period of time, the underlying factors affecting mortality have not changed the character of the mortality significantly.

Once it is decided that a new table will be constructed, the next step is to define the scope of the study. Will the table cover all types of insurance, or will it only be for medically examined cases? Will it be for all cases rated as standard, or will it include all classes of insureds under this category? What period of experience will it cover, the past five years or ten years? These, and many other questions, have to be answered before actual work can begin.

Like any statistical work, much of the value of a mortality table depends on the accuracy of the underlying information. Fortunately, nearly all such material facts used in these investigations are carefully recorded and their accuracy easily verified. Exact dates of birth and death are available, and dates of entry and exit are known.

Although the data are readily available, the labor of obtaining or employing exact information for every case may be prohibitive. It is important to note that, in an investigation of this type, approximations are made from choice, and not from necessity. Sampling techniques, therefore, may be employed to lessen the volume of work, but, with the advent of electronic data processing machines, the trend is to use all available data since the machines can readily handle the volume of work with little increase in cost.

After the selected data are carefully edited, the next step is the choice and application of suitable formulas. The fomulas to be employed will depend upon the method of analyzing the exposures and deaths — policy, calendar, or life year method — and upon the character of the mortality table to be constructed. The selected data, by now properly recorded on cards or tape, are sorted in a manner which will allow the exposure to risk to be obtained in the easiest possible manner.

After sorting and grouping of the data, the result is a crude mortality rate by age. The rate of mortality at age x among a group of lives, who are not subject to any cause of increment nor to any cause of decrement except death, is the probability that a life aged x will die before attaining age $x+1$.

Although theoretically the mortality, q_x, is to reflect death rates of individuals, in the investigation of insured lives, there are four alternative bases for determining q_x:

(1) The death rates among the lives insured;
(2) The rates of termination by death of policies in force;
(3) The rates of termination by death of the amounts insured; and
(4) The rates of termination by death of the premiums involved.

It is incumbent upon the actuary to select the basis he wants and to make proper adjustments in the monetary values thus derived.

The crude mortality rates will then have to be graduated, because a series of observed mortality rates will be found to contain irregularities which are not a feature of the true, underlying rates of mortality. In addition, from a practical point of view, a series of fluctuating mortality rates will create many problems in premium and reserve calculations; in other words, the crude rates need be "adjusted" or "graduated."

Graduation is defined as the process of securing from an irregular series of observed values of a continuous variable a smooth regular series of values consistent in a general way with the observed series of values. This smooth series, or series of graduated values, is then taken as a representation of the underlying law which gives rise to the series of observed values.

Graduation is characterized by two essential qualities: (1) smoothness and (2) fit, or consistency with the observed data. The graduated series should be smooth as compared with the ungraduated series, but it should be consistent with the indications of the ungraduated series.

There are many methods of graduation: graphic method, interpolation method, adjusted-average methods, difference-equation method, and graduation by mathematical formulas.

The graphic method of graduation originated from the graphic method of representing the observed values. Although it is commonly employed, it is more complex than usually thought of. Essentially, it involves visual adjustment of the ungraduated series, by effecting a regular curve among the plotted points, but not necessarily through any particular one of them. (For those who are interested, Mr. T. B. Sprague had a very interesting and complete article entitled "The Graphic Method of Adjusting Mortality Tables, etc." which can be found in the Journal of the Institute of Actuaries, Volume XXVI, page 77.)

Under the interpolation method, the graduated series is obtained by interpolating between special points selected.

These special points are referred to as pivotal points, and the interpolating curve segments are made either to pass through these pivotal points, or, in the modified-interpolation methods, to pass close by them. The most frequently used formulas are the King's formula, Karup-King formula, Shovelton's formula, Jenkin's fifth-difference modified osculatory formula, Buchannan's formula on osculatory interpolation by central differences, Sheppard's formula, and Everett's formula.

The adjusted-average methods include two sets of formulas — linear-compound formulas and summation formulas. Both these formulas are a generalization of an averaging process, the term u_x of the graduated series is expressed as a symmetrical average, or linear compound, of the 2n + 1 ungraduated values, u''_{x+n} to u''_{x-n}, to which it is central. The effect of averaging is to redistribute the error in a particular ungraduated term over the surrounding terms, thereby permitting positive and negative errors partially to offset one another. The formulas found in this connection are the Wolfenden formula, Woolhouse's formula, Higham's formula, Hardy's formula, Karup's formula, Spencer's 21-term formula, and Kenchington's 27-term formula.

Graduation by difference-equation method can be said to be the joint work of Whittaker and Henderson; thus difference-equation formulas are also referred to as Whittaker-Henderson formulas. The method is founded on the formulation of an analytical expression measuring the combination of smoothness and fit, where smoothness is measured by the sum of the squares of the second differences of the graduated values, and closeness of fit is measured by the sum of the squares of the differences between the graduated and the ungraduated mortality rates.

Graduation by mathematical formulas was developed mainly outside the field of actuarial mathematics. There is a great variety of curves which may be used in representing different types of statistical data, such as the curves of Karl Pearson, and the curve systems of Gram-Charlier, Poisson, and

Fourier. However, the curves of greatest interest to the actuary in treating mortality rates are Gompertz' and Makeham's curves, which were developed in the quest for a basic underlying "law" of mortality. Their use has made possible the application of the principle of uniform seniority, which greatly simplifies the calculaton of joint life functions.

A graduated series of mortality rates may then be the final product, that is, a mortality table, ready to be used with or without further modifications. Projections, however, are common, in order to take into consideration trends in mortality, that is, a measure of the mortality to be experienced in the future. Provided that some fairly definite trend is discernible in the past rates, there is no reason why an extension should not be attempted, either by the use of mathematical formulas or by some other method. An example of a widely used table with projections is the Group Annuitants' Table of 1951, which has three different sets of projections.

The other phases of actuarial work also require the extensive use of statistics. An actuary's judgment, decision, and action depend to a large extent on the facts and figures presented. To describe actuarial statistics in detail would be tó describe every aspect of an actuary's work. In this present fast pace of insurance business, this would almost be impossible. But he is essentially an applied statistician, one who utilizes his knowledge in his administration of life insurance business. He, nevertheless, is a practical man — for he occasionally (?) allows himself to be influenced by "non-statistical" considerations, such as the one commonly referred to as "field competition."